

Name: _____ Student ID: _____

INVENTORY MANAGEMENT

MOCK EXAM

EBM026A05 2016–2017.2A FEB RuG

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Read me:

- The exam has a duration of 2 hours.
- The exam has 6 questions, for a total of 100 points.
- The exam is open book. That is, you are allowed to use the textbook Factory Physics (hardcover or print on paper) during the exam, but nothing else.
- We cannot not assign points when your answer is only partly correct, vague, illegibly written, lacks motivation, or not to-the-point.
- You can use the backside of the pages as scrap paper or to continue your answers, if necessary.
- You can also use a calculator that is compliant with the rules as set out by FEB, i.e. no programmable calculators or cell phones.

1. A firm is considering two options to procure a certain product. First option is to produce it in-house. (10p)
 In this case, the procurement (production) cost is \$1.23 per unit and there is a production setup cost of \$20 per each production run. The production rate is 500 rolls per day. The second option is to buy the product from a contractor. In this case, the procurement cost is \$1.26 per item and there is a fixed charge of \$3 per order. In either case, company has an inventory carrying charge of %0.24 per year (i.e. if the procurement cost is \$1 then the holding cost is \$1 · 0.24 = \$0.24 per year). Demand for this product is very stable over time and it is 10,000 items per year. The company is determined to cover all the demand. Which option should the company choose to minimize costs?

Solution: If the company produces its own product, the annual cost will be:

$$\begin{aligned}
 f(Q^*) &= \sqrt{2AD\alpha c(1 - D/P)} + cD \\
 &= \sqrt{2 \cdot 20 \cdot 10000 \cdot 0.24 \cdot 1.23 \cdot \left(1 - \frac{10000}{500 \cdot 365}\right)} + 1.23 \cdot 10,000 = 12634.08.
 \end{aligned}$$

If the company purchases the product, the annual cost will be:

$$\begin{aligned}
 f(Q^*) &= \sqrt{2AD\alpha c} + cD \\
 &= \sqrt{2 \cdot 3 \cdot 10000 \cdot 0.24 \cdot 1.26} + 1.26 \cdot 10000 = 12734.70.
 \end{aligned}$$

Thus, the company should produce the product.

2. Consider the average cost function of the EOQ problem (procurement cost is assumed to be zero)

$$f(Q) = KD/Q + hQ/2$$

where h is the holding cost, D is demand rate, and A is the fixed ordering cost. Let Q^* be the optimal order quantity minimizing $f(Q)$.

- (a) Write an expression for $f(Q^*)$ by means of A , D , and h . (5p)

Solution: We know $Q^* = \sqrt{2AD/h}$. Thus,

$$f(Q^*) = \frac{AD}{Q^*} + \frac{hQ^*}{2} = \frac{AD}{\sqrt{2AD/h}} + \frac{h\sqrt{2AD/h}}{2} = \frac{h\sqrt{2AD/h}}{2} + \frac{h\sqrt{2AD/h}}{2} = \sqrt{2ADh}$$

- (b) Write an expression for the optimal length of the replenishment cycle T^* by means of A , D , and h . (5p)

Solution: Clearly,

$$T^* = \frac{Q^*}{D} = \frac{\sqrt{2AD/h}}{D} = \sqrt{\frac{2A}{Dh}}$$

3. Consider a dynamic lot sizing problem with fixed ordering cost $A = \$50$, holding cost $h = \$1$, and demands over the 4-period planning horizon $D = \{40, 10, 30, 20\}$. Ignore unit procurement costs. Implement the Silver-Meal algorithm to find the order quantity in the first period (not a complete replenishment plan). (10p)

Solution: The average cost per period, assuming that the current period is a and an order is placed to cover the demand in periods $\{n, \dots, n + a\}$ is

$$f(n, a) = \frac{A + \sum_{k=0}^a khD_{n+k}}{a + 1}.$$

We are only interested in the first period ordering decision. Therefore, we compute $f(1, a)$ for different values of a starting from 0 and stop when $f(1, a)$ is larger than $f(1, a - 1)$.

$$\begin{aligned} f(1, 0) &= \frac{50 + \sum_{k=0}^0 k \cdot 1 \cdot D_{1+k}}{0 + 1} = \frac{50 + 0 \cdot 1 \cdot 40}{1} = 50 \\ f(1, 1) &= \frac{50 + \sum_{k=0}^1 k \cdot 1 \cdot D_{1+k}}{1 + 1} = \frac{50 + 0 \cdot 1 \cdot 40 + 1 \cdot 1 \cdot 10}{2} = 30 \\ f(1, 2) &= \frac{50 + \sum_{k=0}^2 k \cdot 1 \cdot D_{1+k}}{2 + 1} = \frac{50 + 0 \cdot 1 \cdot 40 + 1 \cdot 1 \cdot 10 + 2 \cdot 1 \cdot 30}{3} = 40 \end{aligned}$$

Because $f(1, 2)$ is larger than $f(1, 1)$ we stop. We can conclude that we should place an order to cover demands in periods $\{1, 2\}$. This leads to an order quantity of $D_1 + D_2 = 40 + 10 = 50$.

4. Consider a newsvendor problem with the following parameters; unit procurement cost $c = 15$, selling price $p = 30$, and salvage value $s = 10$.

- (a) Assume that the demand D is discrete and uniformly distributed in $[45, 54]$. That is $\Pr\{D = 45\} = \Pr\{D = 46\} = \dots = \Pr\{D = 54\} = 0.1$. Compute the optimal order quantity maximizing the expected profit. (10p)

Solution: We first compute the overage cost h and the shortage cost b . We have $h = c - s = 15 - 10 = 5$ and $b = p - c = 30 - 15 = 15$. We know that

$$F(Q^*) = \min \left\{ Q \mid F(Q) \geq \frac{b}{h + b} \right\} = \min \{ Q \mid F(Q) \geq 0.75 \}$$

Therefore, we need to find the smallest order quantity such that the probability of not stocking-out is greater than or equal to %75. The demand is discrete and uniformly distributed in [45, 54]. Therefore, the distribution function $f(x)$ and cumulative distribution function $F(x)$ are as follows.

x	45	46	47	48	49	50	51	52	53	54
$f(x)$	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
$F(x)$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

It is clear that, in order to attain a non-stockout probability larger than %75, we need to order at least 52 units.

- (b) Compute the expected number of sales, salvages, stock-outs, and profits for the optimal order quantity. (10p)

Solution: We can write (among many other ways) the expected number of sales as $E \min\{Q, D\}$, salvages as $E(Q - D)^+$, and stock-outs as $E(Q - D)^-$.

Because the optimal order quantity is 52; we can compute these as follows:

$$\begin{aligned} E \min\{Q^*, D\} &= \sum_{x=45}^{54} (\min\{52, x\}f(x)) = \left(\sum_{x=45}^{52} x + \sum_{x=53}^{54} 52 \right) \cdot 0.1 \\ &= ((45 + \dots + 52) + 2 \cdot 52) \cdot 0.1 = 49.2 \end{aligned}$$

$$\begin{aligned} E(Q^* - D)^- &= \sum_{x=45}^{54} ((52 - D)^- f(x)) = \sum_{x=53}^{54} ((x - 52)f(x)) = \left(\sum_{x=53}^{54} (x - 52) \right) \cdot 0.1 \\ &= (53 - 52) \cdot 0.1 + (54 - 52) \cdot 0.1 = 0.3 \end{aligned}$$

$$\begin{aligned} E(Q^* - D)^+ &= E(Q^* - D) + E(Q^* - D)^- = Q^* - E D + E(Q^* - D)^- \\ &= 52 - \sum_{x=45}^{54} (xf(x)) + 0.3 = 52 - \left(\sum_{x=45}^{54} x \right) 0.1 + 0.3 \\ &= 52 - (45 + 46 + \dots + 54) \cdot 0.1 + 0.3 = 2.8 \end{aligned}$$

The expected profit function is

$$P(Q) = -cQ + p E \min\{Q, D\} + s(Q - E \min\{Q, D\}).$$

If we plug in Q^* and $E \min\{Q^*, D\}$; we have

$$\begin{aligned} P(Q^*) &= -cQ^* + p E \min\{Q^*, D\} + s(Q^* - E \min\{Q^*, D\}) \\ &= -15 \cdot 52 + 30 \cdot 49.2 + 10 \cdot (52 - 49.2) = 724. \end{aligned}$$

5. Consider a supermarket that uses a (Q, r) -policy to control its inventory of a particular item. The lead time L is positive and constant.

- (a) Formulate a set of recursions to simulate the system over time by making use of the notation provided below. (10p)

D_n the demand during period n

IP_n the inventory position at the end of period n

IL_n the inventory level at the end of period n

Q_n the order quantity in period n

Solution: Three recursions are sufficient.

1. The order quantity in period n is Q if the inventory position at the end of the previous period is smaller than or equal to r , and zero otherwise:

$$Q_n = Q \mathbb{1}\{IP_{n-1} \leq r\}$$

2. The inventory level at the end of period n equals the inventory level at the end of the previous period plus the orders received in period n (placed in period $n - L$) minus the demand in period n :

$$IL_n = IL_{n-1} + Q_{n-L} - D_n$$

3. The inventory position at the end of period n is the inventory position at the end of the previous period plus the order quantity in period n minus the demand in period n :

$$IP_n = IP_{n-1} + Q_n - D_n$$

- (b) By using the recursions you derived, simulate the system over four periods using the following parameters; $r = 3$, $Q = 4$, and $L = 1$. Assume that initial inventory position and inventory level are $IP_0 = 6$ and $IL_0 = 6$ (note that there is no outstanding orders at the outset) and demand over four periods are $D_1 = 4, D_2 = 4, D_3 = 2, D_4 = 1$. Compute IP_n and IL_n as well as order quantities Q_n for $n = 1, \dots, 4$. (10p)

Solution: If we plug in the data into recursions; we obtain the following:

$$n = 1: \quad Q_1 = Q \mathbf{1}\{IP_0 \leq r\} = 4 \cdot \mathbf{1}\{6 \leq 3\} = 0$$

$$IL_1 = IL_0 + Q_0 - D_1 = 6 + 0 - 4 = 2$$

$$IP_1 = IP_0 + Q_1 - D_1 = 6 + 0 - 4 = 2$$

$$n = 2: \quad Q_2 = Q \mathbf{1}\{IP_1 \leq r\} = 4 \cdot \mathbf{1}\{2 \leq 3\} = 4$$

$$IL_2 = IL_1 + Q_1 - D_2 = 2 + 0 - 4 = -2$$

$$IP_2 = IP_1 + Q_2 - D_2 = 2 + 4 - 4 = 2$$

$$n = 3: \quad Q_3 = Q \mathbf{1}\{IP_2 \leq r\} = 4 \cdot \mathbf{1}\{2 \leq 3\} = 4$$

$$IL_3 = IL_2 + Q_2 - D_3 = -2 + 4 - 2 = 0$$

$$IP_3 = IP_2 + Q_3 - D_3 = 2 + 4 - 2 = 4$$

$$n = 4: \quad Q_4 = Q \mathbf{1}\{IP_3 \leq r\} = 4 \cdot \mathbf{1}\{4 \leq 3\} = 0$$

$$IL_4 = IL_3 + Q_3 - D_4 = 0 + 4 - 1 = 3$$

$$IP_4 = IP_3 + Q_4 - D_4 = 4 + 0 - 1 = 3$$

- (c) Compute the average on-hand inventory level and average backorders based on the results you obtained in the simulation. (5p)

Solution: The on-hand inventory level at the end of period n equals IL_n^+ . The average over four periods is

$$\frac{\sum_{n=1}^4 IL_n^+}{4} = \frac{2 + 0 + 0 + 3}{4} = 1.25.$$

The backorder level at the end of period n equals IL_n^- . The average over four periods is

$$\frac{\sum_{n=1}^4 IL_n^-}{4} = \frac{0 + 2 + 0 + 0}{4} = 0.5.$$

- (d) Compute the α -service level (non-stockout probability) and β -service level (fill rate) of the system based on your simulation results. (5p)

Solution: The α -service level is the probability of not stocking out. In our simulation experiment we have four periods, therefore we have four observations on stocking-out or not stocking out. The non-stockout probability can thus be computed as

$$\frac{\sum_{n=1}^4 \mathbf{1}\{IL_n < 0\}}{4} = \frac{1 + 0 + 1 + 1}{4} = 0.75.$$

The β -service level is one minus the ratio of expected shortage to expected demand. In our simulation experiment we have four periods, therefore we have four observations on shortages and demands.

The expected shortage is

$$\frac{\sum_{n=1}^4 IL_n^-}{4} = \frac{0 + 2 + 0 + 0}{4} = 0.5.$$

The expected demand is

$$\frac{\sum_{n=1}^4 D_n}{4} = \frac{4 + 4 + 2 + 1}{4} = 2.75.$$

Therefore, β -service level is $1 - \frac{0.5}{2.75} = 1 - 0.1818 = 0.8182$.

6. An ATM (automated teller machine, i.e., a money machine) is replenished regularly with ‘new’ money. The replenishment works like this: a person takes out the container with money out of the ATM, and replaces it by a new full container.

(a) What shorthand would you use to describe this policy, and why?

(10p)

Solution: This is a periodic-review basestock policy with no lead-time, as money is replenished up to the same level (full container) every time the container is replaced.

Note: This policy is also known as the (R, S) policy where R is the length of the regular replenishment intervals and S is the order up-to level.

- (b) The bank wants to provide a high quality of service to its customers. To that end, the managerial team aims at organizing a replenishment schedule such that the probability of an ATM being out-of-money at any replenishment epoch is below %1. Based on the historical data, it is believed that the daily demand for money at a particular ATM is normally distributed with mean \$9,700 and standard deviation \$3,200. The money container can hold \$90,000. How frequent (in days) this ATM should be visited in order to guarantee the service quality target?

(10p)

Hint: Recall that if D_1, \dots, D_m be m normally distributed random variables with mean and standard deviation values $(\mu_1, \sigma_1), \dots, (\mu_m, \sigma_m)$. Then, their sum $D = D_1 + \dots + D_m$ is also normally

distributed with mean $\mu = \mu_1 + \mu_2 + \dots + \mu_m$ and standard deviation $\sqrt{\sigma_1^2 + \sigma_2^2 + \dots + \sigma_m^2}$.

On the last page of the exam, you will find a table (the so-called z -table) that provides values of the standard normal cumulative distribution function evaluated at different z values (from 0 to 3). This table can be used to evaluate the cumulative distribution function of any normally distributed variable. Let x be a normally distributed random variable with mean $\mu = 8$ and standard deviation $\sigma = 3$. The probability of x being less than or equal to 10 can be obtained from the z table as follows. First compute $z = \frac{10-\mu}{\sigma} = \frac{10-8}{3} \approx 0.67$. Then, look at intersection of the row with 0.6 (first two digits of z) and the column with 0.07 (the third digit of z). For this example we obtain %74.86.

Solution: Let us assume for now that the ATM is visited every n days. Then demand over the replenishment interval is the sum of demands over n days and it is normally distributed with mean $n\mu$ and standard deviation $\sqrt{n}\mu$. Let D^n denote this random variable. The service level criteria specifies a non-stockout probability for each replenishment interval. Because the container holds \$80,000, we need to make sure that the probability of D^n being less than or equal to 80000 is above %99.

For any replenishment interval n this value can easily be obtained through the standard normal cumulative distribution function. In Excel, this function is readily available (i.e. NORMSDIST(z)). But here we will use a z -table instead.

We start from $n = 1$. We compute the mean $n\mu$, standard deviation $\sqrt{n}\sigma$, and corresponding $z = \frac{80000-n\mu}{\sqrt{n}\sigma}$. Then, we find the non-stockout probability $F(z)$ from the z -table. We keep on increasing n and repeating this as long as $F(z)$ is above 0.99. The results are reported below.

n	z	$F(z)$
1	25.09	1.0000
2	15.60	1.0000
3	10.99	1.0000
4	8.00	1.0000
5	5.80	1.0000
6	4.06	1.0000
7	2.61	0.9955
8	1.37	0.9147

It is clear that the ATM should be visited at least every 7 days to guarantee the prescribed service quality, as going for the 8th day reduce the non-stockout probability from %95.55 to %91.47 all at once.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990